MOTOR QUALITY FACTOR
And Why Kq Matters to You (If you like cool motors)!

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“Motor Quality Factor” is the ratio of torque to the square root of power needed to generate that torque. This is measured at zero motor speed, but affects the motor performance across the speed range. Similar sized motors may require almost 20x the heating to produce the same holding torque. The higher the quality factor, the less power is being dissipated in the resistive losses \((P = I^2 \cdot R)\) of the motor for a given torque. The Motor Quality factor, Kq, gives a good method to compare various motors regardless of winding choices. Kq may be easily related to the motor torque constant and the motor winding resistances:

\[
Kq = \frac{\text{Torque}}{\sqrt{\text{Power}}} = \frac{Kt \cdot I}{\sqrt{I^2 \cdot R}} = \frac{Kt}{\sqrt{R}}
\]

With Torque in Nm, Torque constant, Kt is in Nm/A, and winding resistance R is in ohms.

The units of Kq are \(\text{Newton} \cdot \text{meter}/\sqrt{\text{Watts}}\)

It is interesting that Kq remains constant with different voltage windings of the same motor, if the winding fill factor remains constant. A motor rated for twice the voltage would operate with one half the current. \(\Rightarrow\) the torque constant, Kt would double, and the resistance would increase by a factor of 4.

To see how the waste power in the motor is related to Motor quality, the equation for Kq can be rearranged: \(P = \left(\frac{\text{Torque}}{Kq}\right)^2\) For a given torque, doubling the motor quality Kq results in a factory of four less resistive heating for the same amount of torque being generated!

Hybrid servos, with their high pole count, have a very high Kq compared to low pole count traditional servos. We compared a 34HC-1 hybrid servo motor to a low pole count conventional servo of approximately the same size. The torque constants favor the Hybrid servo almost a factor of 2, while the differences in winding resistances again favors the hybrid servo by a factor of almost 6. The resulting Motor Quality factor Kq is 4.3 times higher for the hybrid servo.

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<tr>
<th>Measurement</th>
<th>A34HC-1</th>
<th>X34XXX</th>
<th>Ratio</th>
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<tbody>
<tr>
<td>Kt (Nm/A)</td>
<td>0.214</td>
<td>0.12</td>
<td>1.78</td>
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<tr>
<td>R (ohms)</td>
<td>0.06</td>
<td>.355</td>
<td>.169</td>
</tr>
<tr>
<td>Kq</td>
<td>.873</td>
<td>.201</td>
<td>4.33</td>
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It is necessary to remember that the power dissipated varies as \(1/Kq^2\), so the X34XXX motor, with a Kq that is 4.3x smaller will dissipate almost 18 times the power to produce the same torque. To put this into perspective, at 1 Nm, this is 24.5W vs 1.3w; at 4.85Nm, this is **577W for the low pole count motor vs 31W for the hybrid servo**, thus the amount of time the low pole count X34XXX motor can hold this torque level is quite limited.
Comparison of the best-case efficiency for the low pole count motor versus measured for the hybrid servo, looking at 700 RPM, 5.4Nm or 393W mechanical out. The X34XXX has a resistive loss of approximately 700W (not including driver), and efficiency of 36%. The Hybrid servo has losses of approximately 176 W (including the driver) with a measured efficiency (including the driver) of approximately 69%!

Note that the loss ratio between the motors is not still at the 18:1 ratio, as the Hybrid servo has field weakened the motor to extend its speed-torque range. The field weakening effectively reduces the backEMF to extend the power curve, which in-turn reduces the Kq value due to the lower effective Kt. (See Field Weakening – What it is and Why use it?)

Comparison of Low pole count to Hybrid servo:

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<thead>
<tr>
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<th>Low Pole Count Servo Motor</th>
<th>Hybrid Servo</th>
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<tr>
<td><strong>Rotor Gap</strong></td>
<td>Large -&gt; higher saturation currents High peak torque for a (very) limited duration</td>
<td>Small -&gt; 100 to 150% of current rating High continuous torque</td>
</tr>
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<td><strong>Magnetic design</strong></td>
<td>Face Magnets typical: Not as able to weaken the field</td>
<td>Internal Permanent Magnet: The intervening Iron can be used to reduce the magnetic strength, called field weakening</td>
</tr>
<tr>
<td><strong>Stator Windings</strong></td>
<td>Typically have larger winding area Causes greater winding resistance which lowers Kq</td>
<td>Compact pole windings Low winding resistance for a given Kt</td>
</tr>
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<td><strong>Efficiency</strong></td>
<td>Must be running the motor near its peak speed to get good efficiency. At lower speeds, much greater resistive losses cause much lower efficiency</td>
<td>By use of field weakening, the efficiency can be kept high over a wide speed range</td>
</tr>
<tr>
<td><strong>Holding Power</strong></td>
<td>Lower Kt and higher resistance dissipated much more power, limiting high torque to a very brief period</td>
<td>High continuous torque due to high Kt and low resistance.</td>
</tr>
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</table>
Below is the actual Measured efficiency of a 34HC-1 motor and driver versus speed.

You can see the efficiency passes 60 percent at 300 RPM and remains over 60% until 2500 RPM, an 8 to 1 speed range!
Comparison of QCI-X23C-3 to low pole count integrated servo motor

Both motors are operated from 48vDC, almost identical size.

Direct drive applications, such as belt drives and lead screws typically operate in 250-2000 RPM range, while the low pole count motors are optimized for 5000-8000 RPM due to the lower Kv (backEMF) available with the lower pole count motors with a reasonable winding resistance.

The Hybrid servo has more continuous torque until 2000 RPM, more peak torque through 1500 RPM.

Comparing the output power available between these two motors (see next page), the high initial torque of the hybrid servo provides significantly higher power up to 1500 RPM, with many times the continuous power in the direct drive range of 500-1000 RPM. The Low pole count motor has power capability for these motors for speeds above 2000 RPM. However, the efficiency of the hybrid servo is many times higher in the direct drive 500-1500 RPM speed range, allowing the hybrid servo to operate with much less heating. This speed range is important as it covers the maximum speed for most of the common lead screw applications, limited either by the nut ratings or by the maximum shaft speed to avoid failure for all but the shortest or fattest lead screws.

The Low pole count servo motor has a peak power efficiency of only 18% at 1000 RPM, and a continuous torque efficiency of around 27% as compared to the hybrid servo at 62% DC in to Shaft-out efficiency. The 230% to 340% higher efficiency in this speed range helps keep the hybrid servo motor operating much cooler.

For applications where heating, efficiency, or sustained high torque are required, the Hybrid servo has distinct advantages!
Mechanical Power Versus Speed

Mechanical Power Versus Speed

Efficiency Versus Speed
Field Weakening – What it is and Why use it?

Internal Magnet Synchronous Motors – also called Interior Magnet or Buried Magnet – place the magnet inside of the rotor with soft magnetic material between the magnet and the face of the rotor. With the hybrid motor, a single large disk magnet is used to produce the 100 poles (50 pole pairs) by placing 50 teeth each on the two rotor pole caps that surround the magnet, rather than having to place 100 individual magnets, simplifying construction. (Note that while internal magnet synchronous motor designs exist for lower pole count motors, they are not yet as common as face mounted magnets.)

Field weakening injects a current into the windings that is not in phase with the backEMF (in quadrature). If this current is of the proper phasing and amplitude, it can induce a field into the iron between the magnet and the gap such that the magnet field strength in the gap is weakened. The result is a smaller torque constant and a smaller backEMF.

**Why Weaken the field?** The smaller backEMF extends the speed over which the motor can produce usable torque – keeping the shaft output power approximately constant over a significant speed range. If the torque constant which varies as the backEMF constant (can be equal if the proper units are used) is too high, the backEMF of the motor can exceed the input power supply voltage at a critical speed, thwarting the ability to drive the motor.

The high initial Kt (Torque constant) is useful at lower speeds as it increases the available motor torque thus increasing power available and motor efficiency. As the speed is increased and the Kt is reduced by field weakening, the motor can continue to produce nearly constant output power (speed * torque) while keeping a high efficiency of operation over a wide range of motor speeds.
How Do High Pole Count Motors Achieve High Kt?

The backEMF of the motor is from Faraday’s law: \( V = -\frac{N\delta \phi}{\delta t} \)

The voltage equals the number of turns of wire in the winding times the rate of change of the flux passing through the winding. The amplitude of the flux is determined by the strength of the magnet divided by the reluctance of the path, usually dominated by the air gap (avoiding saturating the magnetic material). The rate of change varies with the product of the motor speed and the number of poles. That is a 4-pole motor would only plot out two sine waves per revolution, while a 100-pole motor generates 50 sine waves for that same revolution (assumes sinusoidal back EMF). For the same gap strength, this would produce 25 times the voltage for the same shaft speed! Typically, the higher pole count motors can be designed with smaller magnets while still producing a very high backEMF resulting in a high motor quality Kq.

The above graph portrays the normalized flux of a 4-pole motor to a 100-pole motor as the shaft is rotated. The rate of change of the flux (slope) times the number of turns is what produces the backEMF. The current through the winding overcoming the backEMF is what is transformed into the motor mechanical power. The higher the product of backEMF and current, the higher the instantaneous motor power.