A Ten-Minute Introduction to Digital Filters

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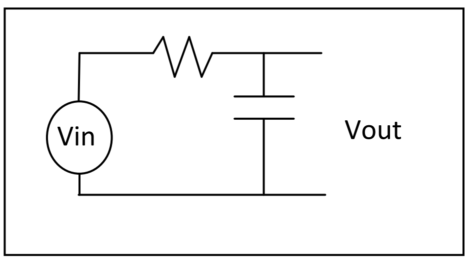
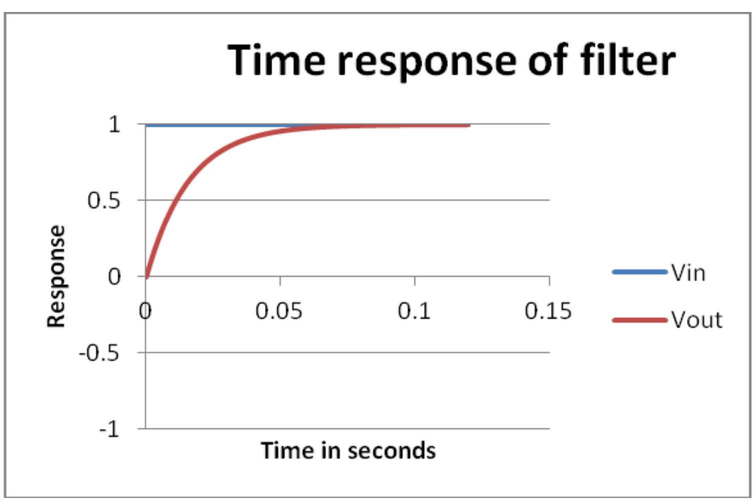
Digital filters are common in communication and control systems, and at least an intuitive understanding of their operation is helpful in their application. This paper is intended to give an arm-chair understanding of their basic operation rather than a full mathematical derivation. Included are links to an excel sheet that implements a simple low pass filter, simulating a single stage RC low pass.

Digital filters may be understood as a real-time simulation of an analog filter circuit. The figure below shows a simple Resistor-Capacitor low pass filter.

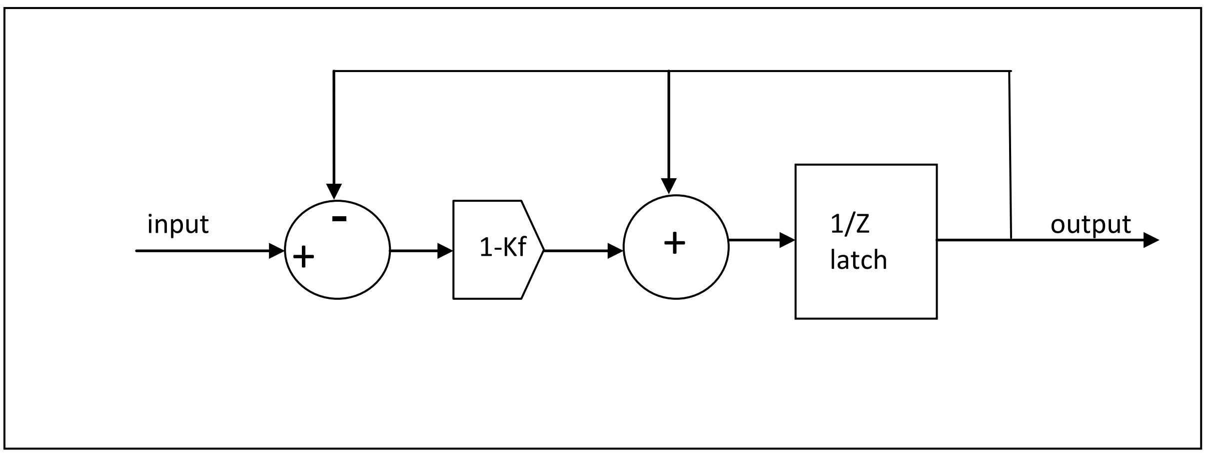


Output vs. Time: The low-pass RC filter can be studied by looking at its time response. Considering the initial state with the capacitor discharged, when a DC input is switched on, the capacitor will charge up, quickly at first, when the voltage difference between the input and the capacitor voltage is large (allowing a higher current to flow into the capacitor to charge it) and then progressively slower and slower as capacitor charges up to the input voltage.

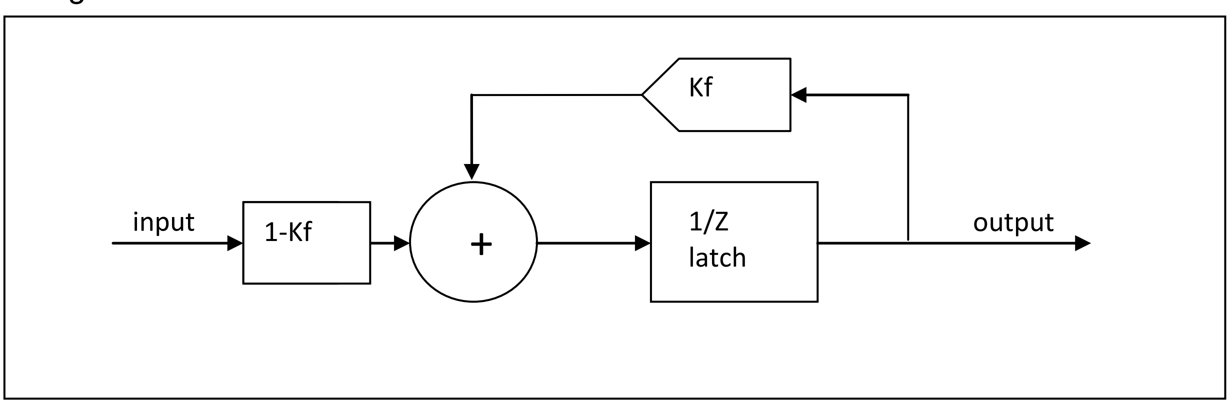


Now consider if the input reverses before the capacitor has fully charged: the output voltage will not have reached the full input voltage level before the input changes direction. The faster the reversals, the smaller the output voltage will be as compared to the input. Thus the higher frequencies are filtered out (reduced) while the low frequencies are allowed to pass.

Digital Filter: The digital filter approximates this response by simulating the interaction of the resistor and capacitor with the input voltage. The voltage across the capacitor proportional to its charge, which is the time summation (Integral) of the current into the capacitor. The capacitor is simulated by the time summation of the simulated current, taken at discrete time steps. As long as the sampling rate is fast compared to the input variations, this is a good approximation. In the computer, this is done by saving the previous "charge" as a value in memory, and adding the new charge (current * time step) to the existing charge each update cycle. The "current" is calculated from the difference in the input voltage and the capacitor voltage. The value stored is updated each sample period. For the QuickSilver Controllers, this is typically every 120 microseconds..



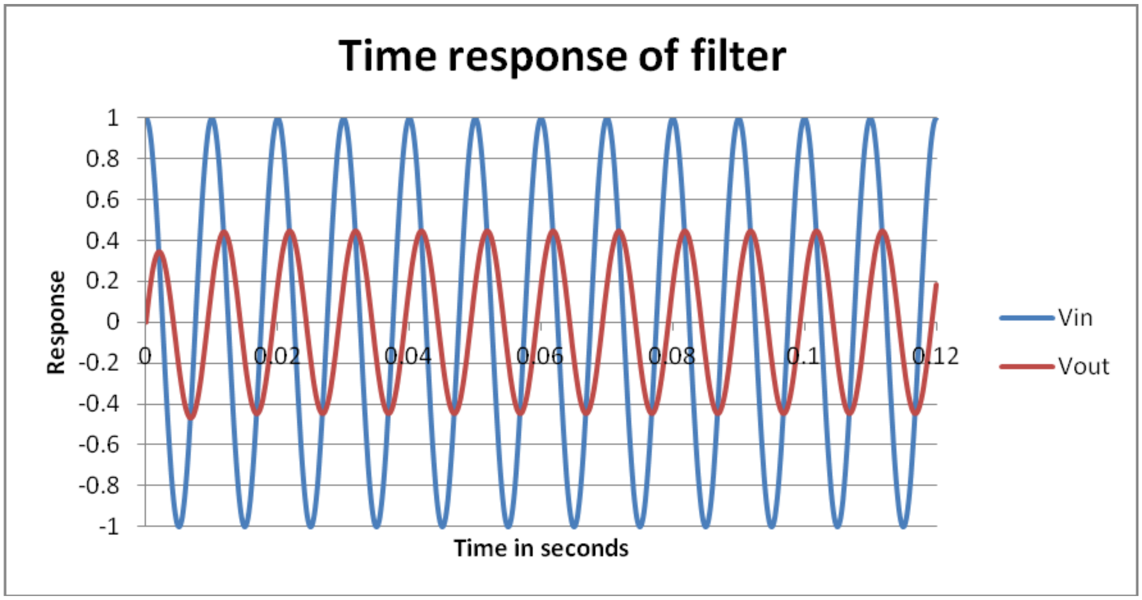
DSP's can multiply as fast as they can add. To speed up the calculations, this is usually re-arranged as shown below:



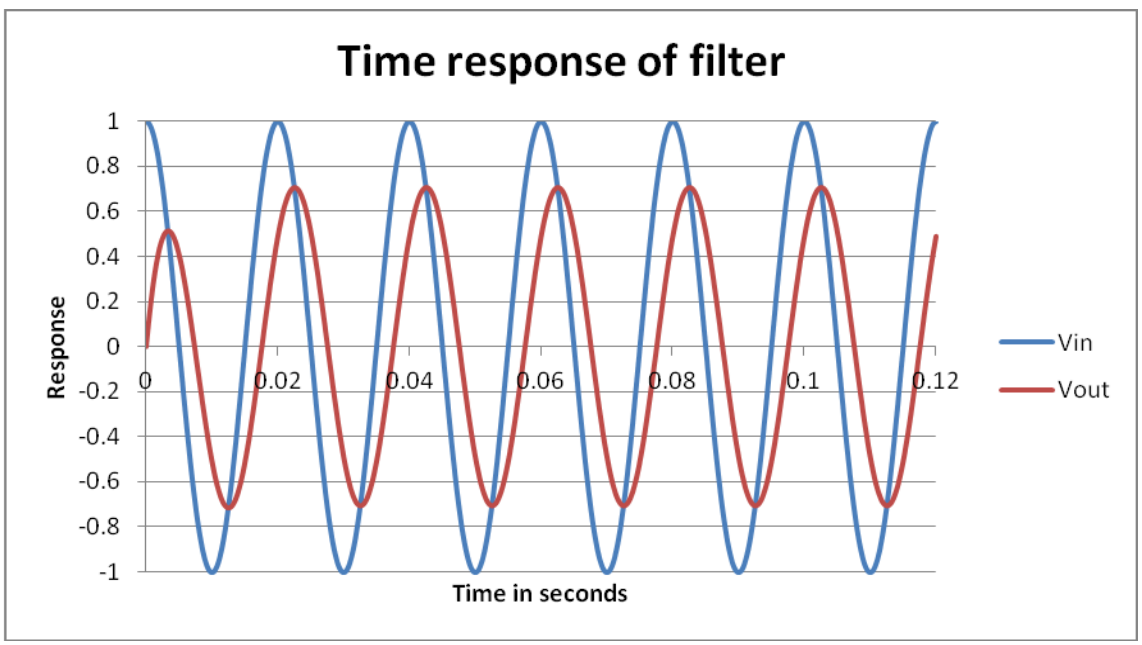
Although the simulation is simplified, it actually produces good results as long as the sampling rate is sufficiently fast compared to the time response of the filter.

Kf is the feedback term, always less than 1, it represents the fraction of the charge remaining on the capacitor after one sample period if the input were zero. It may be observed the new output value will equal the old output value if the input is equal to the old output value. That is the filter has a gain of one for DC inputs.

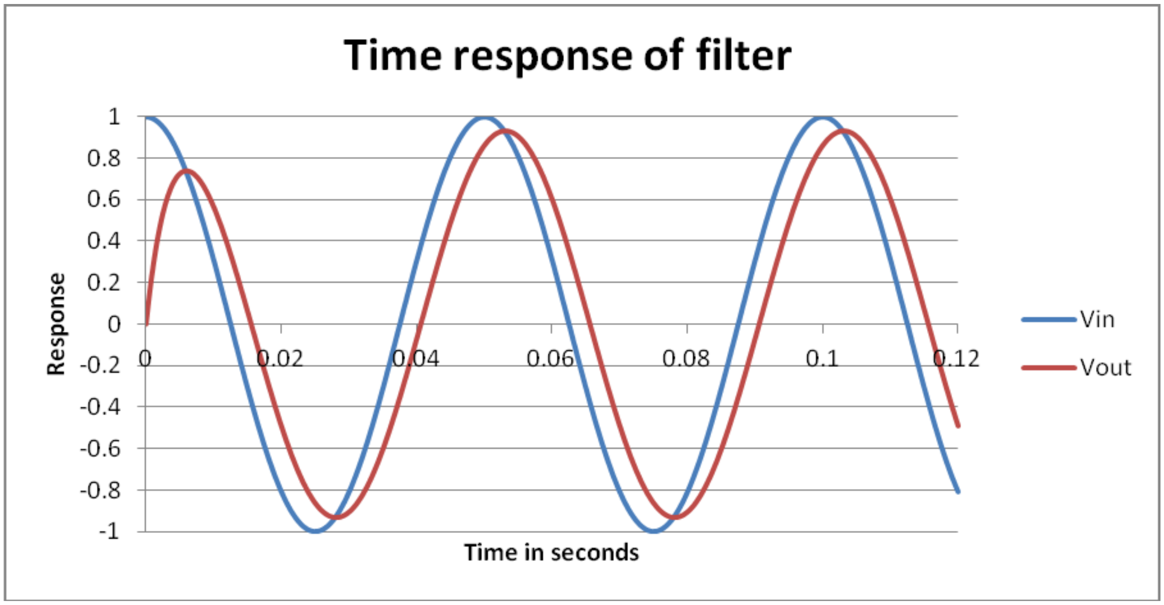
100 Hz input, 50 Hz cutoff selected



50 Hz input, 50 Hz cutoff



20 Hz input, 50 Hz cutoff



You can change the selected cutoff frequency and input frequency. The input frequency should remain below 4kHz for proper operation. The value for Kf is calculated based on the desired cutoff frequency

The simple RC model needs to be adjusted for higher frequencies due to the effects of too few sample points. The value of Kf can be calculated as follows:

$Kf = e^{(-2*Pi*Ffilter*tsample)}$ the filter frequency, Ffilter is (1/RC)/(2*pi) for a frequency in Hertz.

The excel spread sheet (QCI-WP005_DigitalFiltersLesson.xlsx) calculates the streamlined diagram. To see the step response, set the frequency to zero. To see the response to other frequencies, vary the filter cutoff and/or the input frequency. The various charts can be seen by selecting the various worksheets via the tabs at the bottom.

Limits of Simulation: This simple model of the low pass filter works well as long as there are enough samples compared to the changing of the input. For input If the input frequency is more than half the sample frequency, an effect known as "aliasing" will occur, which will cause frequencies in the output that are different from those that were applied to the input (down mixing for those familiar). These are normally not desired and should be avoided.

I hope you enjoyed this 10-minute introduction to digital filters.